THE QUANTUM FLUCTUATIONS OF LIGHT BEAMS

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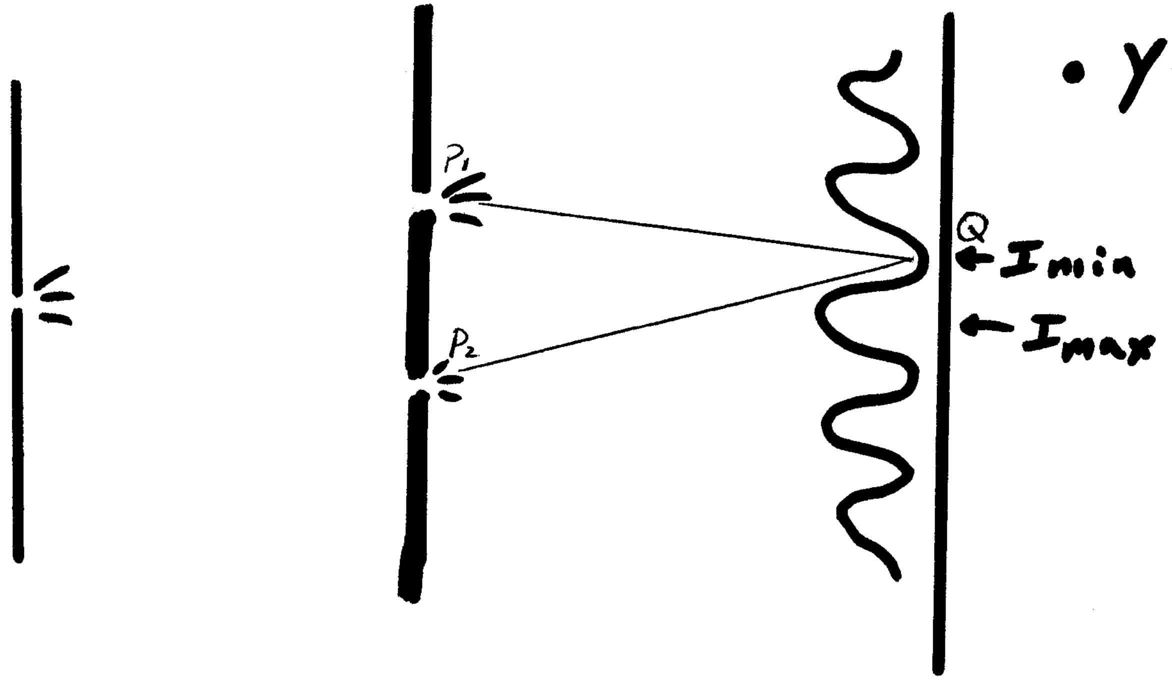
THE FLUCTUATIONS OF LIGHT -- OUTLINE

- CLASSICAL CONCEPT OF "COHERENCE"
  - VISIBILITY OF FRINGES
  - correlation function of FIELD vs. spectrum
- FLUCTUATIONS OF NATURAL LIGHT
  - thermal light
  - Bose-Einstein statistics
  - Hanbury-Brown, Twiss EFFect
- LASER LIGHT & Coherence
  - narrow spectrum vs. P[I]
  - Generalization of "coherence"
- ORIGINS OF QUANTUM STATISTICS OF LIGHT
  - Linear vs. NONLINEAR STIM'D EMISSION
  - Linear superposition - spont. emission
  - Analysis of MODELS

FRINGE VISIBILITY - COHERENCE



- YOUNG'S INTERFERENCE EXPERIMENT

✓ $V = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$

$\equiv$ VISIBILITY OF FRINGE (MICHELSON)

✓ MEASURES: Degree of Coherence

$$E(Q) = a_1 E_1(P_1) + a_2 E_2(P_2)$$

$$I = a_1^2 I_1 + a_2^2 I_2 + 2 a_1 a_2 \,\mathrm{Re}\underbrace{\langle E_1 E_2^* \rangle}$$ ✓MUTUAL COHERENCE FUNCTION

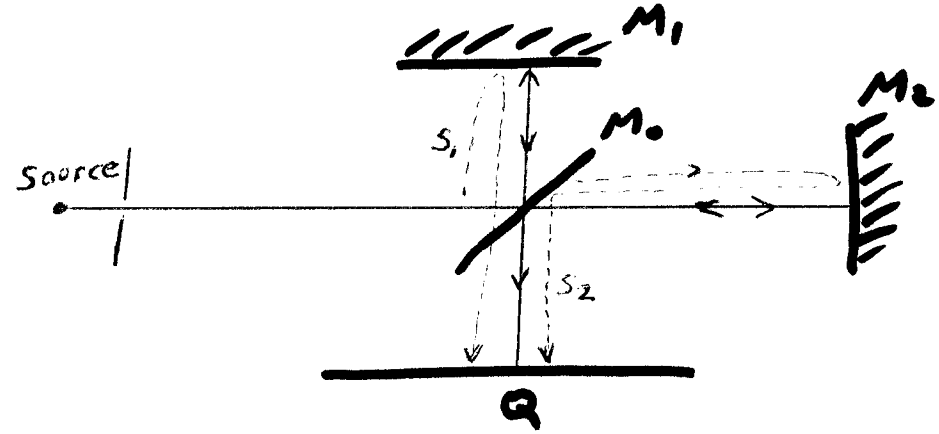
$$= I_1' + I_2' + 2\sqrt{I_1' I_2'}\, |\underbrace{\gamma_{12}(\tau)}| \cos\delta$$

✓COMPLEX DEGREE OF COHERENCE

$$\therefore \quad V = \frac{4\sqrt{I_1' I_2'}}{2(I_1 + I_2)} |\gamma_{12}(\tau)| \xrightarrow[I_1 = I_2]{} |\gamma_{12}(\tau)| \leq 1$$

TEMPORAL COHERENCE

- MICHELSON INTERFEROMETER



- QUASIMONOCHROMATIC LIGHT $\sim \Delta\nu$

$$\Delta S = S_2 - S_1 = c\,\Delta t$$

- Wiener-Khinchine Th'm

$$\gamma_{11}(\tau) \longleftrightarrow g(\nu)$$

$$\tau_c \sim \frac{1}{\Delta\nu}$$

- coherence time:

$$\gamma_{11}(\tau) \approx 1; \forall\ \tau < \tau_c$$

$\tau_c > \Delta t \Rightarrow$ GOOD FRINGES at Q

- Coherence length

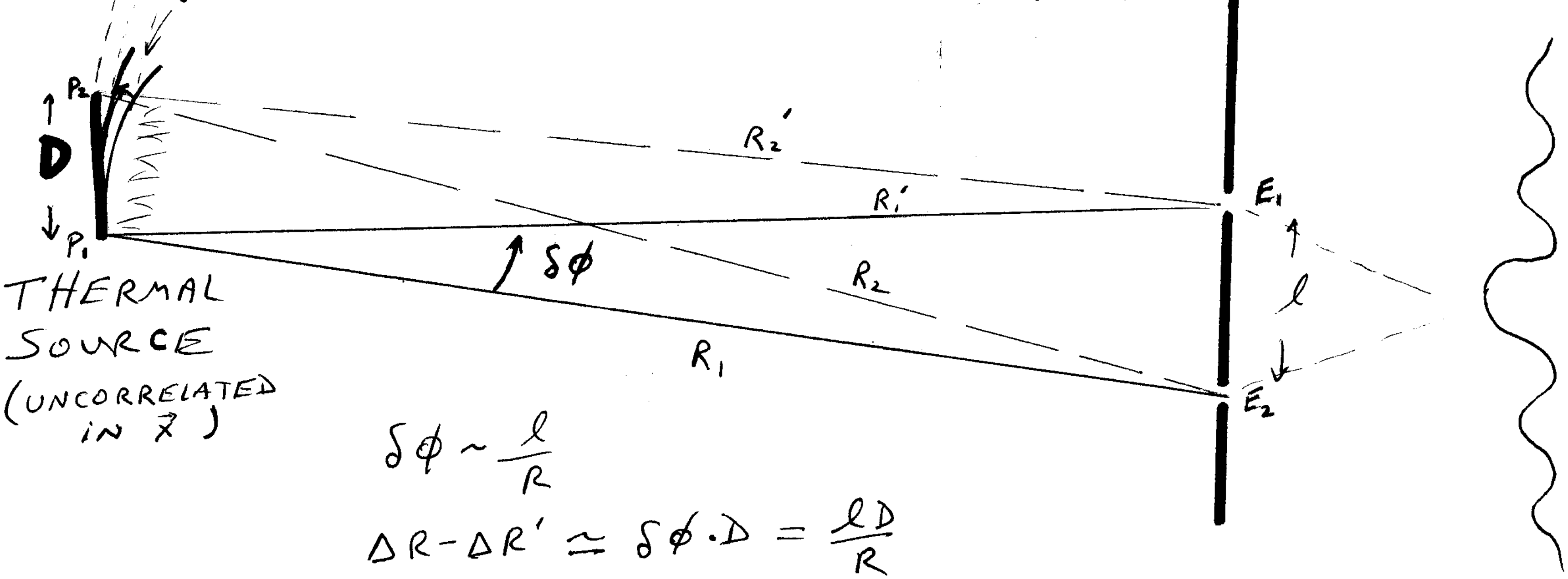
$$l_c = c\tau_c = c/\Delta\nu$$

- $g(\nu) \longrightarrow$ Visibility of Fringes

SPATIAL COHERENCE

- van Cittert - Zernike Theorem

"SPATIAL" WHITE NOISE $\delta(x_1 - x_2)$ → $\boxed{\dfrac{\text{Propag'n}}{\text{SPATIAL FILTER}}}$ → SPATIALLY CORRELATED (coherent) $\gamma(x_1, x_2, 0)$



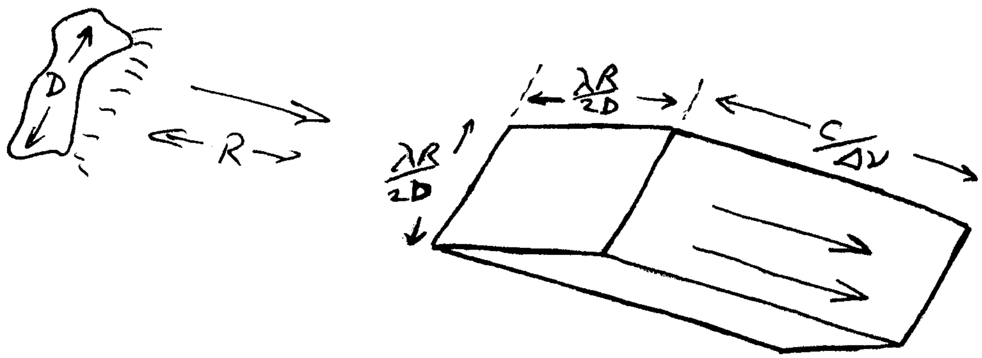
$$\delta\phi \sim \frac{l}{R}$$

$$\Delta R - \Delta R' \simeq \delta\phi \cdot D = \frac{lD}{R}$$

$$\lesssim \frac{\lambda}{2} \quad \Rightarrow \quad E_1 \simeq E_2$$

$$l = \frac{R\lambda}{2D} \simeq \text{TRANSVERSE COHERENCE LENGTH}$$

COHERENCE VOLUME



$$V_{coh} = \left(\frac{\lambda R}{2D}\right)^2 \frac{c}{\Delta\nu} = \frac{\lambda^4 R^2}{\Delta\lambda D^2}$$

$$\delta \equiv \frac{\bar{n}_{photons}}{V_{coh.}} = \frac{1}{e^{h\nu/kT}-1} << 1$$ THERMAL LIGHT

$\delta >> 1$ LASER LIGHT

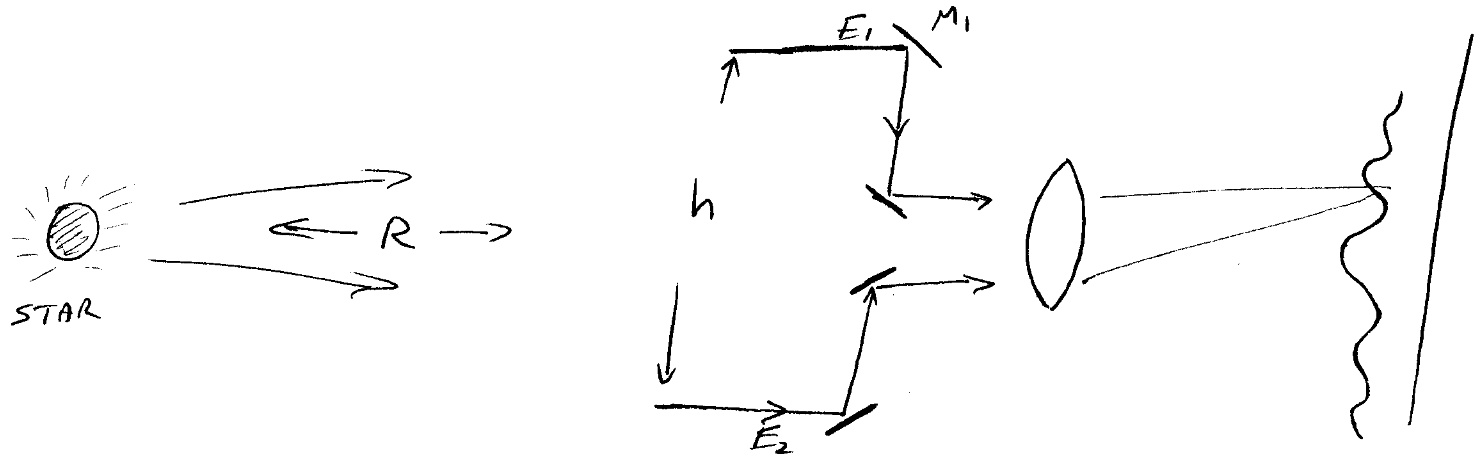
COMPARE: "DETECTOR" VOLUME

$$V_{det} \approx A_{det} \cdot \frac{c}{\Delta\nu_{det}}$$ ← detector bandwidth

- UNLESS $V_{det} \leq V_{coh}$
  - $\rightarrow A_{det} \leq A_{coh}$
  - $\rightarrow \frac{1}{\Delta\nu_{det}} \leq \frac{1}{\Delta\nu_{rad}}$

measurements ARE "LONG" TIME "LARGE" AREA AVERAGES
⇒ MASKS TRUE STATISTICS OF LIGHT

MICHELSON STELLAR INTERFEROMETER



- TRANSVERSE SPATIAL COHERENCE $\sim \frac{\lambda R}{2D}$
- Stellar Diam $\approx \frac{1.22\,\bar{\lambda} R}{h}$ $\uparrow h$: FRINGES VIS $\rightarrow \emptyset$

DRAWBACKS : PHASE FLUCTUATIONS FROM TURBULENCE IN AIR, etc LIMIT MAX. h.

FLUCTUATIONS OF NATURAL LIGHT

- superposition OF RANDOM INDEPENDENT FIELDS
- RANDOM WALK IN PLANE – CENTRAL LIMIT TH'M

$\Longrightarrow$ GAUSSIAN DISTRIB'N OF FIELD AMPL.

$$\therefore \quad P(I)\,dI = \int_0^{2\pi} d\phi \frac{r\,dr\, e^{-r^2/2\sigma^2}}{2\pi\sigma^2} = \frac{d(r^2)}{2\sigma^2} e^{-r^2/2\sigma^2} = \frac{e^{-I/\bar{I}}}{\bar{I}}\,dI$$

$\Longrightarrow$ EXP'L DISTRIB'N OF INTENSITY

- $\langle I^n \rangle = n!\,\langle I \rangle^n$

  e.g. $\langle I^2 \rangle = 2\langle I \rangle^2$ $\Longleftarrow$ (whereas, for const. INTENSITY LIGHT, LIKE LASER $\langle I^2 \rangle = \langle I \rangle^2$)

- $$p_n = \int P(I)\, e^{-\eta I t} \frac{(\eta I t)^n}{n!}\, dI = \frac{\bar{n}^n}{(1+\bar{n})^{n+1}}$$ B.E. (geometric) distrib'n

$$\langle (\Delta n)^2 \rangle = \bar{n}(1+\bar{n}) = \underbrace{\bar{n}}_{\text{particle aspect}} + \underbrace{\bar{n}^2}_{\text{wave aspect}}$$

ASPECTS OF GAUSSIAN RANDOM PROCESSES

- Any Linear Transformation ON Gaussian Process → Gaussian
  - ✓ Propagation in Space → INCREASED COHERENCE AREA
  - ✓ Narrow band (temporal) spectral FILTER → incr. coh. length

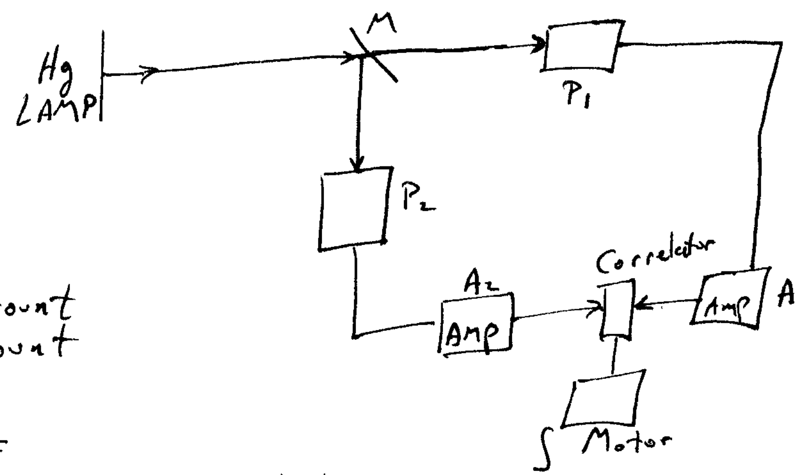
    But $p(I)$ remains neg. exp'l (But wait τ_c for it to change)
    $p(E)$ " Gaussian
    → p_n REMAINS Bose-Einstein

- CHARACTERISTIC OF Thermodynamic EQUILIBRIUM
  e.g. b.b. radiation
  But also of non-equilibrium situations → B.E. stat's.
  Laser → spinning ground glass

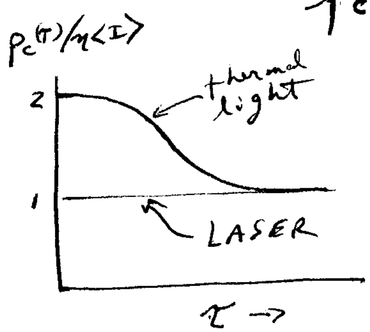
- Completely CHARACTERIZED BY 1st & 2nd MOMENTS
  - ✓ $\langle I(t)\, I(t+\tau)\rangle$ can be written in terms of $\langle E(t)\, E^*(t+\tau)\rangle$
    etc.; MULTI-TIME TOO: $\langle I(t)\, I(t_2)\, I(t_3) \cdots\rangle$
  - ✓ $g(\nu)$ + Wiener-Khinchine ⇒ JUST ABOUT EVERYTHING

HANBURY-BROWN, TWISS EFFECT

- IDEA FOR INTENSITY STELLAR INTERFEROMETER
  - INTENSITY w/i COHERENCE VOLUME
  - Measure CORRELATIONS IN INTENSITY w/i COHERENCE VOLUME
  - thermal photons tend to bunch together
  - FIRST $\langle i_1 i_2 \rangle$, later $\langle n_1 n_2 \rangle$
  - phase lost, no longer problem



Cond'l PROB. of photocount
τ sec's after first count



$$p_c(\tau)\,d\tau =$$

$$= \frac{\eta^2 \langle I(t+\tau) I(t)\rangle\, dt\, d\tau}{\eta \langle I(t)\rangle\, dt}$$

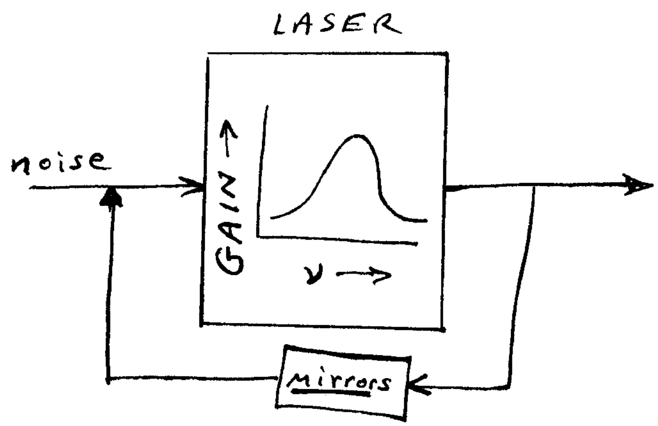
$$= \eta \langle I(t)\rangle \left[1 + |\gamma(\tau)|^2\right]$$

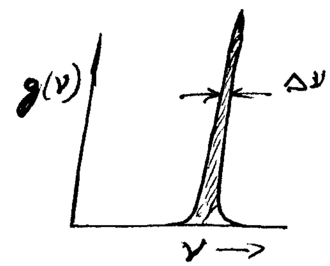
$\rightarrow \eta\, 2 \langle I(t)\rangle$ for $\tau \ll \tau_c$

$\rightarrow \underbrace{\eta \langle I(t)\rangle}_{\text{same as for laser}}$ for $\tau \gg \tau_c$ $\langle I^n\rangle = \langle I\rangle^n$

✓ Polarized LIGHT

FLUCTUATIONS OF LASER LIGHT

- 



$\Leftarrow$ phase diffusion

$E(t) \approx E_0 \, e^{i\omega t + i\phi}$

$\uparrow$ fixed

POSITIVE FEEDBACK + NONLINEARITIES $\Rightarrow$

1) VERY NARROW Linewidth

2) Amplitude is CLAMPED AT I_0

-- NOT THE SAME THING!

-- COULD FILTER THERMAL LIGHT so $\Delta\nu_{th} = \Delta\nu_{\ell}$

BUT $P_{th}[I]$ WOULD STILL BE expl.

$\therefore$ STILL exhibit H-B,T. EFFECT

- OFTEN CALLED COHERENT
  - WHAT IS MEANT ?
  -- CERTAINLY $\gamma_{12}(\tau) \simeq 1$ over LARGE VOLUME ($\Delta\nu$ SMALL)
  -- But could do that with FILTERED THERMAL LIGHT

- $P[I] \simeq \delta(I - I_0)$! $\Rightarrow \langle I^n \rangle = \langle I \rangle^n$

- $|\gamma_{12}(\tau)| \simeq 1 \Rightarrow |\langle E_1(t) E_2^*(t+\tau) \rangle| = \sqrt{I_1}\sqrt{I_2}$ CLASSICAL COHERENCE

- Generalization $\rightarrow$ "HIGHER ORDER" CORRELATION FUNCTIONS FACTOR

LASER LIGHT

- 1963 JAVAN et al : $\frac{\Delta\nu}{\nu_0} = 8/10^{14}$, $\lambda_0 = 1153$ nm   <u>He Ne Laser</u>

  $\Delta t = 4.8 \times 10^{-2}$ s , $\Delta l_c = \underline{\underline{14.4 \times 10^{6} m}}$

- 1973 Chebotayev : $\frac{\Delta\nu}{\nu_0} = \frac{6}{10^6}$ Methane stabilized He Ne

  $\Delta t = 6.4$ s $\Rightarrow \Delta l_c \simeq \underline{\underline{20 \times 10^{8} m}}$!

- $p[I] = \delta(I - I_0) \Longrightarrow p_n = \int P(I)\, e^{-\eta I T} \frac{(\eta I T)^n}{n!}\, dI$ ✓ assumes $T < \tau_c$

  $= e^{-\eta I_0 T} \frac{(\eta I T)^n}{n!}$ ✓ <u>Poisson</u>

  $\therefore \langle (\Delta n)^2 \rangle = \langle n \rangle \longrightarrow$ i.e. <u>photocounts</u> <u>are</u> <u>completely</u> <u>independent</u>

- GENERAL COHERENCE

  $\Rightarrow$ MULTITIME/SPACE PHOTOCOUNTS ARE MUTUALLY INDEPENDENT

  $\langle \Delta n_1 \Delta n_2 \Delta n_3 \cdots \rangle = \langle \Delta n_1 \rangle \langle \Delta n_2 \rangle \cdots$

ROLE OF STIMULATED EMISSION

- IN PRODUCING B.E. STATISTICS, H-B,T ?
- $\exists$ 2 ALTERNATE VIEWPOINTS (SURPRISINGLY)

1) stim. em $\Rightarrow$ photon correlations, B.E. STATISTICS, H-B,T

VS.

2) STIMULATED em. coherently amplifies any initial field
   - no change in nature of photon stat's.
   - except, add'n of $\not\!\!\&$ interference with spontaneous emissions

- Both are somewhat plausible
  - for 1) consider Einstein's deriv. Planck b.b. law
    - if leave OUT STIM. em. N.G.
  - 2) consider laser amplifier far below threshold

Role OF STIM. EMISSION – References

- D.B. SCARL & S.R. Smith
  "TIME CORRELATIONS IN STIM. EMISSION"
  Phys. Rev. A 10, 709-713 (1974)

- L. Mandel
  "STIMULATED EMISSION & PHOTON CORREL'S."
  Phys. Rev. A 14, 2351-2354 (1976)

- Rockower, Abraham & Smith
  "EVOLUTION OF THE QUANTUM STATISTICS OF LIGHT"
  PHYS. REV. A 17, 1100-1112 (1978)

Essentially Four MODELS

- SINGLE ISOLATED ATOM – LOWEST ORDER IN TIME (B.E. not changed by INT.)
- LASER AMPLIFIER (below threshold)
  - theory & exp't difficult to interpret
  - identify spont. & stim. em terms in eqns
- LASER OSCILLATOR
  - STIM. em. has MAJOR ROLE, NO H-B;T.
- Exact (RWA) single atom in $\bar{E}$ field.

→ R. A. & S. (1978)
SOLVE FOR Prob. Generating Function $\langle z^n \rangle$
and QUANTUM CHARACTERISTIC FUNCTION
$Tr(\rho e^{\xi a^+} e^{\eta a^+})$

LASER AMPLIFIER

- $C(\xi,\eta) = e^{\bar{n}_s \xi\eta}\, C_0(\xi e^{\gamma t/2}, \eta e^{\gamma t/2})$; $\bar{n}_s = ASE$

  If initial field is VACUUM, $C_0 = 1$

  – Hence, 1st factor is QUANTUM char Function for Gaussian ASE

  – SECOND Factor $\Rightarrow$ "coherent" Amplification of INPUT FIELD

  i.e. normalized moments are unchanged since

$$\left(\frac{d}{d\xi}\right)^j \left(\frac{d}{d\eta}\right)^k C_0(\xi e^{\gamma t/2}, \eta e^{\gamma t/2}) \Big|_{\xi=\eta=0} = e^{(j+k)\gamma t/2} \langle (a^\dagger)^j a^k \rangle_0$$

$\Longrightarrow$ FINAL FIELD = superposition of TWO FIELDS

FACTORING of $C(\xi,\eta) \Rightarrow$ SIMULTANEOUS INDEP. STOCH. PROCESSES.

$\therefore$ SINCE ASE IS B.E., & SINCE IT IS COHERENTLY AMPLIFIED, IT MUST HAVE ORIGINATED AS B.E.

– Hence STIM. em plays no role in Laser Amplifier STATISTICS, AS ASE "EVOLVING" TOWARDS B.E. as some have claimed.

EXACT SINGLE ISOLATED ATOM

- INTERACTS W. RESONANT MODE OF FIELD
- We calc. gen. function $\langle z^{a^+a} \rangle$
- Assume Atom initially in upper state, field described by density matrix $\rho(0)$.
- If n photons in Field at time t, $\exists$ 2 possibilities
  1) $n-1$ at $t=0$ & atom now in lower level
  2) n at $t=0$ " " " upper "

$$\therefore \quad \rho_n(t) = \rho_{n-1}^{(0)} \, |C_{b,n}(t)|^2 + \rho_n^{(0)} \, |C_{a,n}(t)|^2 .$$

$$|C_{b,n}(t)|^2 = \sin^2(g\sqrt{n}\,t) \qquad |C_{a,n}(t)|^2 = \cos^2[g\sqrt{n+1}\,t]$$

Rabi's "flopping" atom : in Quantized field
- no collisions
- no rate eq'ns

a ___

b ___

SINGLE ATOM - $G(z)$

- MULTIPLY by z^n & Trace

$$G(z,t) = G_0(z) + (z-1)\langle \sin^2[g(n+1)^{1/2}t]\, z^n \rangle$$

$\Rightarrow$ nature of generating function changes in general

∴ even B.E. STATISTICS NOT CONSERVED

- FULL NONLINEAR STIM. EM. <u>DOES</u> CHANGE photon STATISTICS

- SINCE B.E. STATISTICS NOT maintained, Difficult to justify POSITION THAT THIS INTERACTION $\Rightarrow$ B.E. !

CONCLUSION

- In those cases in which stim.em leads TO LINEAR amplific'n. (no change of photon statistics): Linear superpos'n, central limit Th'm, random walk in plane ARE SUFF. ⇒ B.E. STAT'S.

- IN NONLINEAR CASES OF STIM. EM. (laser, or single atom) B.E. STAT'S ARE either not present, or not even preserved